Method of controlling chaos with self-coupled variables and without feedback*

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Abstract A new method for controlling chaos in dynamical systems with self-coupled variables and without feedback is presented. Using this method it is not necessary to trace the evolution of system states or add additional controlling system. Thus, this method is simple and effective in practice. The results of theoretical analyses and numerical computations are obtained using this method.

Keywords: non-feedback, self-coupled, control, chaos.

The systems in real world are generally nonlinear, therefore the phenomena of bifurcation and chaos are universal. Although chaos is a kind of harmful factor under many circumstances, all kinds of unstable periodic and non-periodic states are embedded in the chaotic attractor, which contains very rich information. If some methods can be found, which may transform the chaotic state of system into a stable periodic state, the chaotic system will be turned into the one with many uses. Thus the goal of utilizing chaos will be achieved. Since Ott, Grebogi and York (OGY) first presented a method for controlling chaos by the tiny adjustment of parameters in 1990[1], controlling chaos has already become an active branch in the field of nonlinear science.

So far, researchers have already presented many effective methods for controlling chaos in practice. These methods are primarily divided into two classes: with and without feedback. The former mainly includes the methods of OGY[1], proportional pulse[2], self-adaptive control[3—4], continuous feedback[5], and so on. The latter mainly includes the methods of parameter perturbation[6], coupled system[7], and so on. In non-feedback methods, it is not necessary either to know beforehand certain linearization characteristics of unstable periodic orbits embedded in the chaotic attractor, or to extract the unstable periodic orbits desired to be stable (namely the target orbit), thus they have the advantages that the other methods do not have. However, in some non-feedback methods, such as the coupled system method presented by Kapitaniak[7], one needs to add an additional stable linear system coupled with the original chaotic system in order to control chaos, which makes these methods difficult to be used. The method presented in this paper can overcome this shortcoming, in which the variables of a chaotic dynamical system are directly coupled with themselves instead of an additional linear system for controlling chaos. Applying this method to the two-dimensional discrete Henon map and three-dimensional continuous Rossler equations, satisfactory results have been obtained.

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1 Principle and method of control

Consider a two-dimensional map as follows:

$$\begin{cases} x_{n+1} = f(x_n, y_n), \\ y_{n+1} = g(x_n, y_n), \end{cases} \tag{1}$$

where f and g are two unknown functions. Assuming that the system has an unstable fixed point (x_F, y_F), and

$$\begin{cases} x_F = f(x_F, y_F), \\ y_F = g(x_F, y_F). \end{cases} \tag{2}$$

Adding two self-coupled terms to the right side of the two equations in eq. (1), which turns into

$$\begin{cases} x_{n+1} = f(x_n, y_n) + \varepsilon(y_n - x_n), \\ y_{n+1} = g(x_n, y_n) + \varepsilon(x_n - y_n), \end{cases} \tag{3}$$

where ε is a self-coupled coefficient. Linearizing the above equations, the following equation can be obtained:

$$\begin{pmatrix} x_{n+1} - x_F \\ y_{n+1} - y_F \end{pmatrix} = J \begin{pmatrix} x_n - x_F \\ y_n - y_F \end{pmatrix}. \tag{4}$$

In eq. (4), the Jacobian matrix is

$$J = \begin{pmatrix} f_x - \varepsilon & f_y + \varepsilon \\ g_x + \varepsilon & g_y - \varepsilon \end{pmatrix}, \tag{5}$$

where

$$f_x = \left.\frac{\partial f}{\partial x}\right|_{x_F, y_F}, \quad f_y = \left.\frac{\partial f}{\partial y}\right|_{x_F, y_F}, \quad g_x = \left.\frac{\partial g}{\partial x}\right|_{x_F, y_F}, \quad g_y = \left.\frac{\partial g}{\partial y}\right|_{x_F, y_F}.$$

In order to make (x_{n+1}, y_{n+1}) closer to (x_F, y_F) than (x_n, y_n), the two characteristic values of J

$$\lambda_{\pm} = \frac{f_x + g_y - 2\varepsilon}{2} \pm \frac{\sqrt{(f_x + g_y - 2\varepsilon)^2 - 4[(f_x - \varepsilon)(g_y - \varepsilon) - (f_y + \varepsilon)(g_x + \varepsilon)]}}{2} \tag{6}$$

should satisfy the condition $|\lambda_{\pm}| < 1$. The range of value ε, in which the system will converge, can be determined by eq. (6).

2 Applications of the controlling method in discrete and continuous system

2.1 Control application in Henon map

Two-dimensional Henon map can be of the form:

$$\begin{cases} x_{n+1} = 1 - ax_n^2 + by_n, \\ y_{n+1} = x_n, \end{cases} \tag{7}$$

for which the unstable fixed points are

$$x_F = y_F = \frac{(b-1) + \sqrt{(b-1)^2 + 4a}}{2a}. \tag{8}$$

When $a = 0.5$, $b = 0.3$, the system is stable with period 2. When $a = 1.4$, $b = 0.3$, the system displays chaotic behavior as shown on the left of figs. 1 (a) and (b), respectively. Coupling the two variables x and y with system (7), we have

$$\begin{cases} x_{n+1} = 1 - ax_n^2 + by_n + \varepsilon(y_n - x_n), \\ y_{n+1} = x_n + \varepsilon(x_n - y_n). \end{cases} \tag{9}$$

From eq. (6), the range of value ε can be determined. Let $\varepsilon = -0.85$, the control starts when the iteration step number $n = 2\,500$, the system rapidly converges to the unstable fixed point as shown on the right of figs. 1 (a) and (b), respectively.

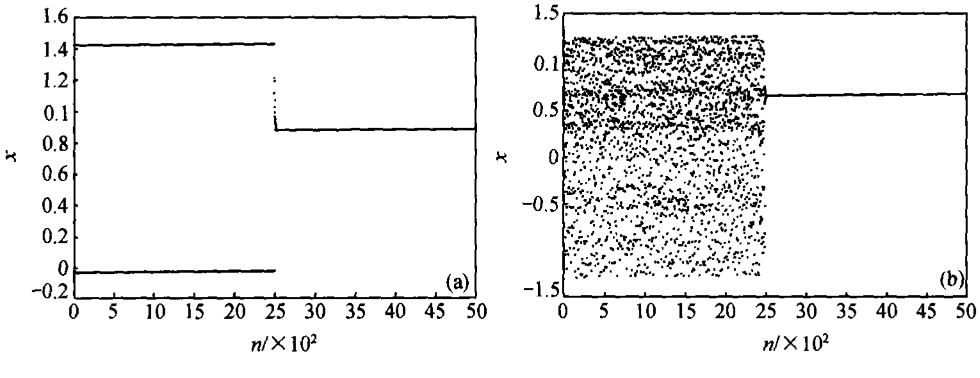

Fig. 1. The controlled Henon map. (a) From period 2 to the unstable fixed point; (b) from chaotic state to the unstable fixed point.

2.2 Control application in Rossler equations

Rossler equations, a three-dimensional autonomous differential dynamical system, are of the form:

$$\begin{cases} \dfrac{dx}{dt} = -y - z, \\ \dfrac{dy}{dt} = x + ay, \\ \dfrac{dz}{dt} = b + xz - \mu z. \end{cases} \tag{10}$$

The unstable fixed points are

$$x_F = \frac{\mu \pm \sqrt{\mu^2 - 4ab}}{2}, \quad y_F = \frac{-\mu \mp \sqrt{\mu^2 - 4ab}}{2a}, \quad z_F = \frac{\mu \pm \sqrt{\mu^2 - 4ab}}{2a}. \tag{11}$$

Let the initial values $x_0 = y_0 = z_0 = 0$, and the time step $h = 0.001$. Numerically integrating the equations, with the step number of integrating $n = 10^5$ and the parameters $a = b = 0.2$, $\mu = 3.7$, the system shows stable state of motion with period 2 on $x - y$ phase plane as shown in fig. 2 (a). When $a = b = 0.2$, $\mu = 5.7$, the system displays obvious chaotic behavior as shown in fig. 3(a). Coupling variable y with variable z, eqs. (10) is turned into

$$\begin{cases} \dfrac{dx}{dt} = -y - z, \\ \dfrac{dy}{dt} = x + ay + \varepsilon(z - y), \\ \dfrac{dz}{dt} = b + xz - \mu z + \varepsilon(y - z). \end{cases} \tag{12}$$

Let $\varepsilon = 0.3$, and the control begins when the step number of integrating $n = 5 \times 10^4$, the chaotic behavior of system is rapidly suppressed, and the system converges to a solitary fixed point on $x - y$ phase plane. The results are shown in figs. 2 and 3, respectively.

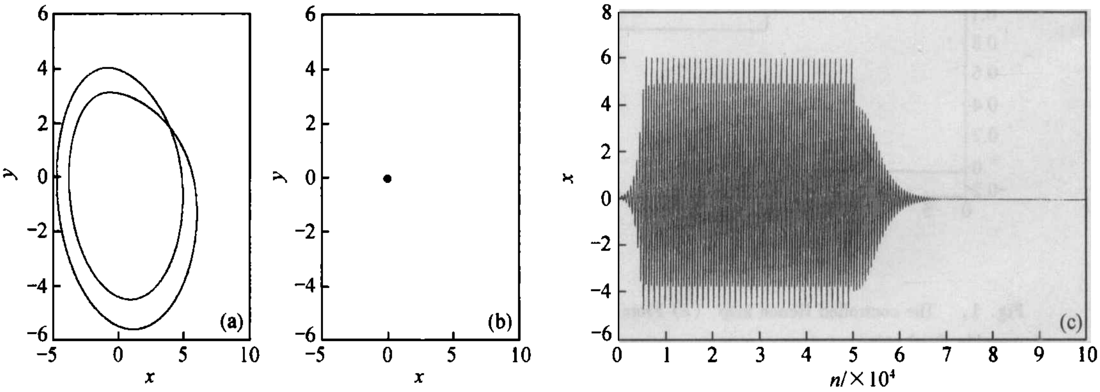

Fig. 2. The controlled Rossler equations. (a) Period 2; (b) the unstable fixed point; (c) the variation of the time series of variable x.

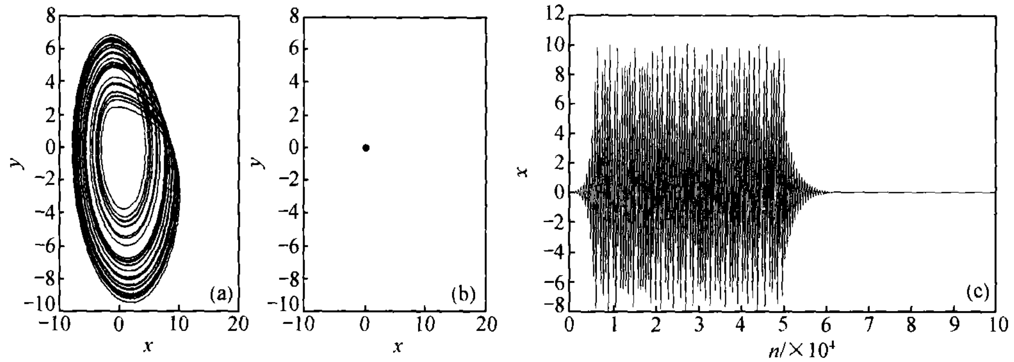

Fig. 3. The controlled Rossler equations. (a) Chaotic motion; (b) the unstable fixed point; (c) the variation of the time series of variable x.

2.3 Control application in Lorenz equations

The well-known Lorenz equations are

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x), \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = (r - z)x - y, \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = xy - bz. \end{cases} \tag{13}$$

When the parameters $\sigma = 10$, $r = 28$, $b = 8/3$ the system displays obvious chaotic behavior with three unstable fixed points:

$$O(0,0,0), \quad P^{+}(x_F, y_F, z_F), \quad P^{-}(-x_F, -y_F, z_F), \tag{14}$$

where $x_F = y_F = \sqrt{b(r-1)}$, $z_F = r - 1$. Let the initial values $x_0 = 3$, $y_0 = 2$, $z_0 = 4$, the time step $h = 0.01$. Numerically integrating the equations with the step number of integrating $n = 10^5$, and coupling variable y with variable z, the equations are turned into

$$\begin{cases} \dfrac{dx}{dt} = \sigma(y - x), \\ \dfrac{dy}{dt} = (r - z)x - y + \varepsilon(z - y), \\ \dfrac{dz}{dt} = xy - bz + \varepsilon(y - z). \end{cases} \tag{15}$$

Let $\varepsilon = 0.5$, and the control begins when the step number of integrating $n = 5 \times 10^4$, the chaotic behavior of system is rapidly suppressed, and the system converges to a solitary fixed point on $x - z$ phase plane. The result is shown in figure 4.

3 Conclusions

The method of controlling chaos put forward in this paper has the following characteristics.

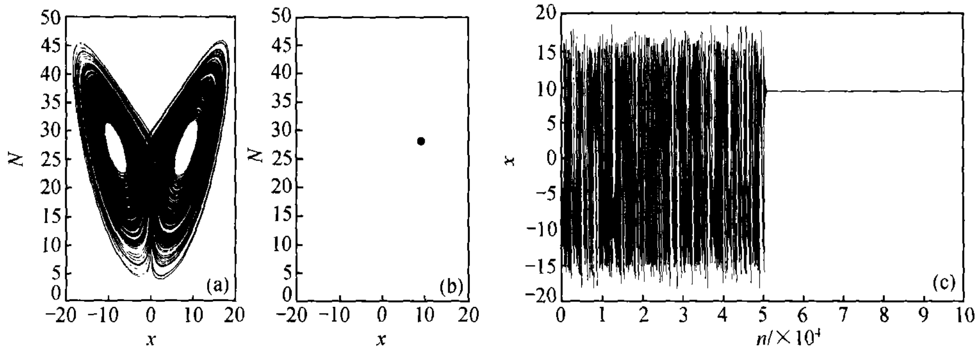

Fig. 4. The controlled Lorenz equations. (a) Chaotic motion; (b) the unstable fixed point; (c) the variation of the time series of variable x.

(i) It is not necessary either to know certain linearization characteristics of unstable periodic orbits, or to extract the target orbits from unstable periodic orbits and measure them. Additional control system is still not needed.

(ii) The state of motion of system can be turned from chaotic into regular periodic by coupling its own variables in the differential equations directly.

(iii) If the derivatives f_x, f_y, g_x and g_y can be extracted from time series[8], the range of value ε, in which the system will converge, can be determined. It is not necessary to know the equations of system beforehand, thus the application of this method can be broadened greatly.

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